





Search 1: Concepts	start time:
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This activity introduces some ideas and techniques used in artificial intelligence (AI) to search for possible solutions to problems. In many disciplines, **real problems** can be complicated and involve special cases, boundaries, etc. Thus, we often start with **toy problems** that are easier to describe, analyze, or solve.

Before you start, complete the form below to assign a role to each member.
If you have 3 people, combine Speaker & Reflector.

Team	Date
Team Roles	Team Member
Recorder: records all answers & questions, and provides copies to team & facilitator.	
Speaker: talks to facilitator and other teams.	
Manager: keeps track of time and makes sure everyone contributes appropriately.	
Reflector: considers how the team could work and learn more effectively.	

This activity uses several icons:

-  is a **key** question; you should have a good answer that everyone understands.
-  is an **optional** question; you should skip it if you are behind schedule.
-  is a **pause**; check with the instructor before you continue.
-  is a **stop**; wait for other teams to catch up before you continue.

Reminders:

1. *Note the time whenever your team starts a new section or question.*
2. *Write legibly & neatly so that everyone can read & understand your responses.*

It's so much easier to suggest solutions when you don't know too much about the problem.

-- Malcolm Forbes



(16 min) A. 8-Puzzles	start time:
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1. (2 min) **8-puzzles** have a 3x3 board with 8 sliding tiles and 1 space (see example below). The goal is to slide one tile at a time until they form a familiar picture or sequence. How many moves are possible if the space is in:

a.	the center?	
b.	a corner?	
c.	the middle of a side?	

2. (4 min) For the initial layout shown below, fill in blank boards to show each layout that is:
 a. **1 move away** from the initial layout.
 b. **2 moves away** from the initial layout (you may not need all of the boards).

Initial Layout		<table><tr><td>6</td><td>8</td><td>4</td></tr><tr><td>3</td><td>1</td><td></td></tr><tr><td>2</td><td>7</td><td>5</td></tr></table>	6	8	4	3	1		2	7	5																																														
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3. (2 min) Which layout(s) above are repeated, and how many times are they repeated?

4. (2 min) If you added each layout that is 3 moves away, which layouts would be repeated?

5. (2 min) 🗝️ In complete sentences, explain why it might be a good idea to avoid repeated layouts when searching for a solution.

6. (2 min) ❓ The same idea can be adapted to other size boards. For example, a **15-Puzzle** has 15 pieces on a 4x4 board, and a **3-Puzzle** has 3 pieces on a 2x2 board.


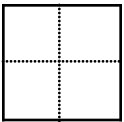
a.	For a 15-Puzzle, what is the minimum number of possible moves?	
b.	what is the maximum number of possible moves?	
c.	For a 3-Puzzle, what is the minimum number of possible moves?	
d.	what is the maximum number of possible moves?	

👋 Check with the instructor before you continue.

(10 min) B. Normal Magic Squares	start time:
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1. (4 min) A **normal magic square (NMS)** of order N is a set of distinct integers $1 \dots N^2$ laid out in a N -by- N square such that all rows, all columns, and both diagonals sum to the **magic constant** $M = N(N^2+1)/2$.

a.	What is M for a square of order 3?	
b.	Is square (b) at right a NMS?	
c.	Is square (c) at right a NMS?	
d.	Is there a NMS of order 1?	
e.	Is there a NMS of order 2?	

(b)	(c)
8 3 4	2 9 4
1 6 9	7 5 3
5 7 2	6 1 8
(d)	(e)
	

2. (2 min) 🗝️ Explain your answers to 1d and 1e:

3. (2 min) For a **blank square** of order 4:

a.	How many cells (positions) does it have?	
b.	If all cells are empty, how many could get the value 1?	
c.	If 1 is in a cell, how many could get the value 2?	
d.	If 1 & 2 are in cells, how many could get the value 3?	
e.	If 1, 2, 3, & 4 are in cells, how many could get the value 5?	

4. (2 min) Answer these questions with an **expression**, not a **value**. If all cells are empty:

a.	How many ways could 1 & 2 be placed?	
b.	How many ways could 1, 2, 3, & 4 be placed?	
c.	How many ways could all numbers be placed?	

👉 Check with the instructor before you continue.

(10 min) C. 8-Queens Puzzle	start time:
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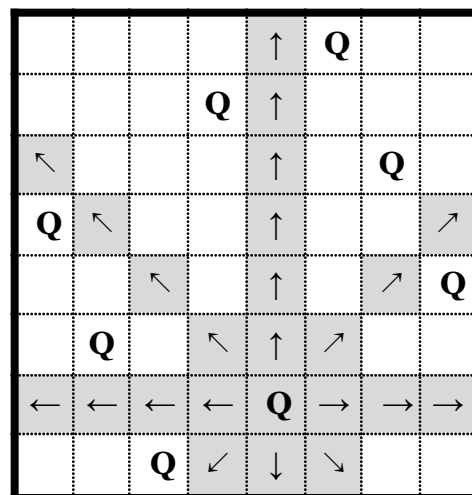
1. (3 min) A chessboard is an 8x8 grid, and each position can contain one chess piece.

How many ways are there to place:

a.	1 piece?	
b.	2 pieces?	
c.	4 pieces?	

2. (2 min) The **queen** piece can move any distance in any direction, as shown by the shaded positions at right. The goal of the **8-Queens Puzzle** is to place 8 queens such that none share a row, column, or diagonal.

a.	In diagram at right, do any queens share a row?	
b.	do any queens share a diagonal?	



3. (2 min) 🗝️ Explain why it is not possible to place 9 queens such that none share a row, column, or diagonal.

4. (3 min) ? These toy problems (*8-Puzzles*, *Normal Magic Squares*, and *8 Queens*) share some characteristics with real-world problems, such as:

- Deciding which items to place on which shelves in a retail store.
- Deciding the times and classroom locations for courses in a high school or college.

Describe key characteristics that all these problems share, and explain what makes the real-world problems more difficult?

Science never solves a problem without creating ten more.

-- George Bernard Shaw

🛑 Wait until the instructor tells you to continue.



(16 min) D. States & Actions	start time:
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Many AI **problems** (or approaches to solve them) have similar structures, and consistent terminology can help us to compare problems and approaches. This terminology may be familiar, if you have studied **finite state automata (FSA)**. Each problem involves some set of **actions** (or **moves**) that change its **state**. Each problem starts from an **initial state** and tries to reach a **goal state**, where it passes a **goal test**. Most problems have one initial state (some have more), and zero, one, or more **goal states**. A problem with no goal states can't be solved.

1. (4 min) Describe the **initial state** and a suitable **goal test** for each problem:

Problem	Initial State	Goal Test
NMS of order N	NxN grid of empty cells	Do all rows, all columns, and both diagonals sum to the magic constant M ?
8-Puzzle		
8-Queens		

2. (2 min) Answer the questions below:

a.	Does a NMS of order 2 have a goal test ?	
b.	a goal state ?	
c.	A 3-Puzzle has the initial state and goal state shown below. Is it possible to reach the goal state from the initial state?	

initial:	goal:								
<table><tr><td></td><td>3</td></tr><tr><td>1</td><td>2</td></tr></table>		3	1	2	<table><tr><td>1</td><td>2</td></tr><tr><td>3</td><td></td></tr></table>	1	2	3	
	3								
1	2								
1	2								
3									

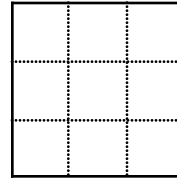
In general, each **state** has a set of **applicable actions**, and each action leads to a **successor** state (also called a **next** state). A **transition model** or **transition function** defines how actions move from one state to another.

3. (2 min) An **8-Puzzle** has at most **4 actions**:

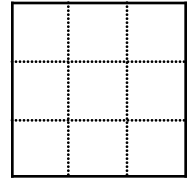
slide left, right, up or down.

At right, show states with only 3 and 2 applicable actions.

a. 3 actions

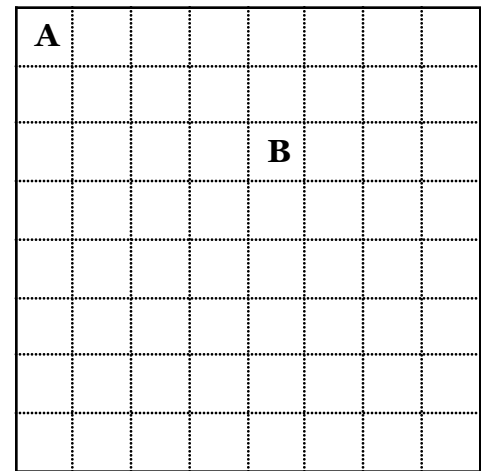


b. 2 actions



4. (2 min) In an **8-Queens Puzzle**, each action makes other actions **inapplicable**.

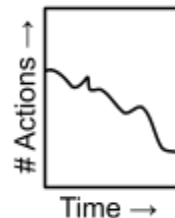
At right, put a 1 in each position blocked by Queen A, and then a 2 in each position blocked by Queen B.



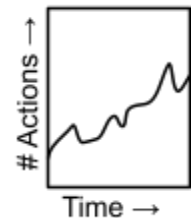
5. (2 min) The number of applicable actions can vary, but there can also be a general trend. Over time, does the number tend to go **up**, **down**, or stay the **same** in:

a.	8-Puzzle	
b.	8-Queens	
c.	Normal Magic Square (NMS)	

goes down



goes up



6. (2 min) 🗝 In general, will it be easier to search states if the number of applicable actions goes **up** or **down** over time? Explain your answer.

👉 Check with the instructor before you continue.


(10 min) E. Paths & State Space	start time:
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1. (2 min) A sequence of states connected by applicable actions is a **path**. In some problems, the challenge is to find a **goal state**; in other problems, the goal state is known or obvious, and the challenge is to find a **goal path**. Categorize each of the following as a **goal state** or **goal path** problem.

a.	8-Puzzle	
b.	8-Queens	
c.	Normal Magic Square	

2. (3 min) Some paths may be better than others, even if they reach the same goal state; thus, we can define a **path cost function** to compare paths. For an **8-Puzzle**:

a.	Would most people prefer a path with 20 actions & states, or 200?	
b.	What is a possible path cost function ?	

3. (2 min)  The initial state, possible actions, and the results of those actions (the **transition model**) together define the **set of all possible states** that can be reached. This is called the **state space**. In complete sentences, describe the state space for **tic-tac-toe**.

4. (2 min)  Some problems have states that cannot be reached through any sequence of actions, and thus are not in the state space. Describe 2 **unreachable states** for tic-tac-toe.

 Wait until the instructor tells you to continue.

(16 min) F. Searching State Space

start
time:

We can think of **state space** as a **map**, where each state is a place on the map. Our goal is to find a set of actions to move from the **initial state** to a **goal state**.

The figure at right is a sample map, with **initial state** IN, **goal states** G1-G3, and up to 4 actions from each state. Each “1” is 1 step away from IN, each “2” is 2 steps away from IN, etc.

							G1		
				3					
			3	2	3				
		3	2	1	2	3			
	3	2	1	IN	1	2	3		
		3	2	1	2	3			
			3	2	3				
				3					G3
G2									

1. (3 min) On the map above, how many states are:

a.	2 steps away from IN?	
b.	3 steps away from IN?	
c.	4 steps away from IN?	

2. (2 min) Would the number of states within 4 steps increase **faster** or **slower**:

a.	if each state had 2 possible actions?	
b.	if each state had 6 possible actions?	

3. (2 min) A map of state space can help us think about ways to search state space. The number of actions on a path is the **path length**.

a.	Which of the 3 goals has the shortest path length from IN?	
b.	Which goal is farthest from IN?	

Any search **problem** can be formally defined by:

- **state space** (initial state, actions, transition model)
- **goal test**
- **path cost function**

Artificial intelligence typically considers search problems with large and complex state spaces, in which an algorithm can only explore a small fraction of the states.

4. (3 min) 🗝️ The table below provides describes general categories of problems. For each description, decide whether solving the problem will require **one** action from each state, **several** actions, or **many** actions (at random).

5. (4 min) 🗝️ Rank the categories in the table below from easiest (1) to hardest (6). If two categories seem equal, give them an equal ranking.

	Description of Problem Category	Approach	Ranking
a.	You know the goal state. You don't know which actions lead towards it.		
b.	You know the goal state. You have clues for which actions lead towards it.		
c.	You don't know the goal state. You have clues for which actions lead towards it.		
d.	You know the goal state. You know which action(s) leads towards it.		
e.	You don't know the goal state. You don't know which actions lead towards it.		
f.	You don't know the goal state. You know which action(s) leads towards it.		

6. (2 min) 🗝️ Describe the principles or insights that you used to rank the categories.

🛑 Wait until the instructor tells you to continue.

(8 min) G. Analysis using Factorial	start time:
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1. (2 min) The product of the sequence $1*2*...*n$ is written as **n!** and called **n factorial**.

Answer each question below with an expression that uses factorial(s) -
do not multiply or divide to get a final value.

a.	Write out the integers in 5!	
b.	Write out the integers in 8!	
c.	Use 5! in an expression for 8!	
d.	Use 5! and 8! in an expression for $6*7*8$	
e.	Write an expression for the number of ways to fill a blank square of order 4 :	

2. (2 min) In the **8-Queens Puzzle**:

a.	How many ways could 8 queens be placed?	
b.	Write the answer to the previous question using 2 factorials	

3. (3 min) In the **general case** of a **blank square** of **order N**.

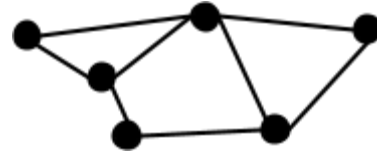
a.	How many cells does it have?	
b.	How many ways could the value 1 be placed?	
c.	How many ways could the values 1 & 2 be placed?	
d.	How many ways could all values be placed?	
e.	How many ways could all but the last 2 values be placed?	
f.	How many ways could all but the last 3 values be placed?	
g.	How many ways could all but the last M values be placed?	
h.	How many ways could the first M of N values be placed?	



Wait until the instructor tells you to continue.

(7 min) H. Traveling Salespersonstart
time:

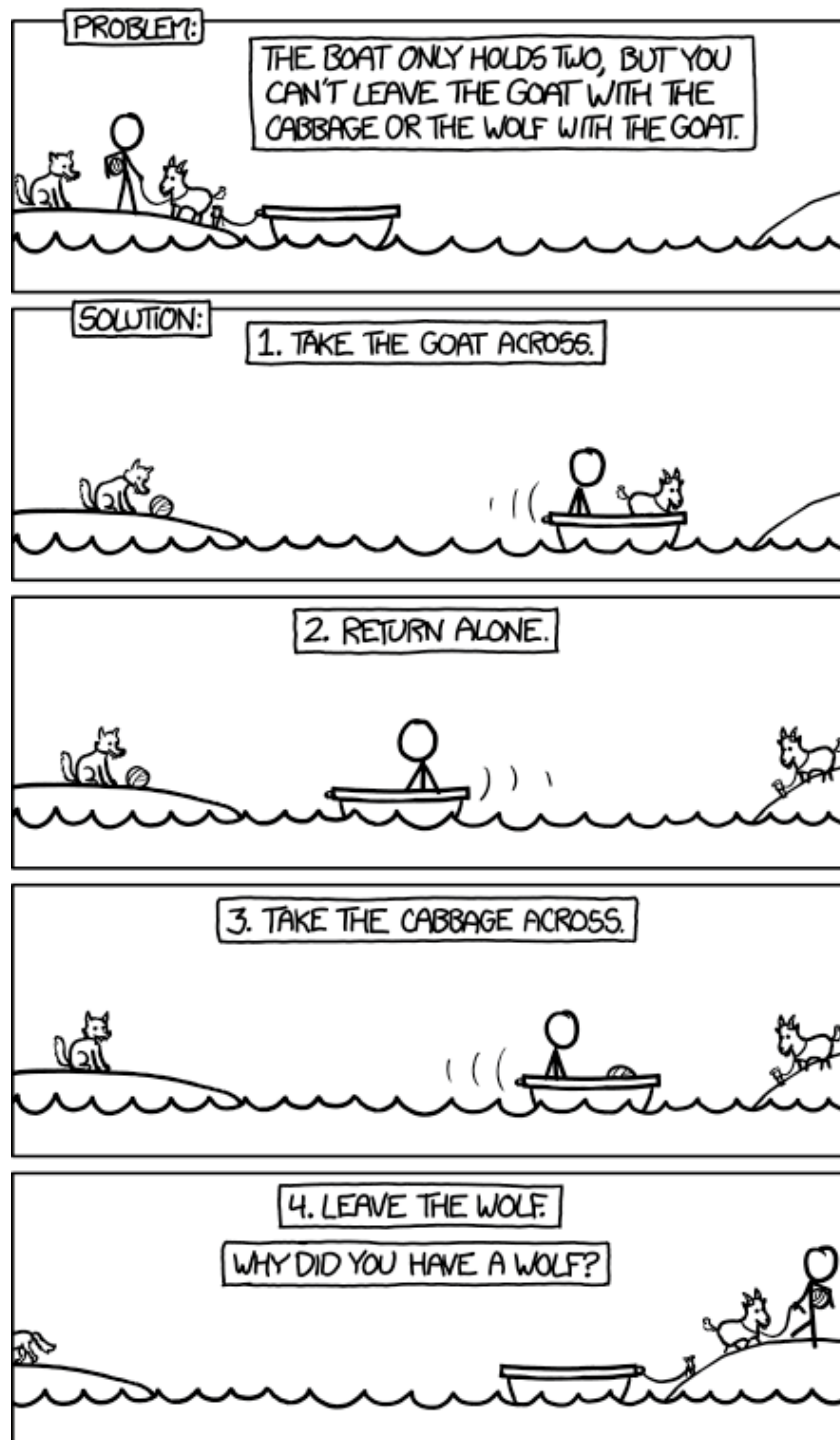
1. (2 min) In the **Traveling Salesperson Problem**, we have a list of cities and the distances between them. The goal is to find the **shortest trip** that visits each city exactly **once** and returns to the starting position. Explain why a non-ideal solution might be acceptable.



2. (2 min) 🗝 Describe how the **path cost function** for **Traveling Salesperson** might depend on distance, time, sales, or expenses (e.g. for meals, hotel rooms, flights).
3. (2 min) ? Why might techniques to solve Traveling Salesperson be of great interest to delivery services (e.g., Amazon, FedEx, UPS)? What other problems are similar?

For every complex problem there is an answer that is clear, simple, and wrong.

-- H. L. Mencken



<http://xkcd.com/1134/>